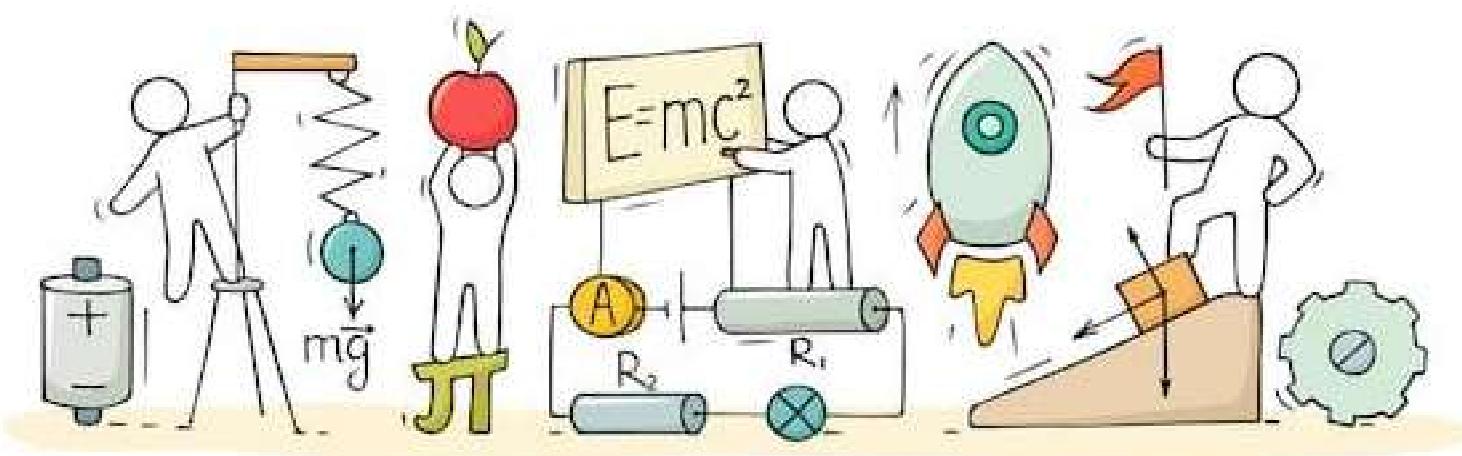
PHYSICS

Chapter 4: Motion in a Plane



Motion in a Plane

Top Formulae

Projectile Motion Thrown at an angle with the horizontal	(a) $y = x\tan\theta - \frac{1}{2}.g.\left[\frac{x}{u\cos\theta}\right]^2$ $\vec{U} = u\cos\theta\,\hat{i}$ $a_x = 0$ $\vec{U}_y = u\sin\theta\,j$ $a_x = -g\hat{j}$ Or $y = x\tan\theta\left[1 - \frac{x}{R}\right]$
	(b) Time to reach maximum height $t = \frac{u\sin\theta}{g} = \frac{u_y}{y}$
	(c) Time of flight $T = \frac{2u\sin\theta}{g} = \frac{2u_y}{y}$
	(d) Horizontal range $R = \frac{u^2\sin2\theta}{g} = u_x \times T$
	(e) Maximum height $H_{max} = \frac{u^2\sin2\theta}{2g} = \frac{u_y^2}{2a_y}$
	(f) Horizontal velocity at any time $v_x = u\cos\theta$
	(g) Vertical component of velocity at any time $v_y = u\sin\theta - gt$
	(h) Resultant velocity $\vec{v} = v_x\hat{i} + v_y\hat{j}$ $\vec{v} = u\cos\theta\,\hat{j} + (u\sin\theta - gt)\hat{j}$ $v = \lvert\vec{v}\rvert = \sqrt{u^2 + g^2t^2 - 2ugt\sin\theta}$ and $\tan\alpha = \frac{v_y}{v_x}$

	General Result For maximum range θ = 45° $R_{max} = \frac{u^2}{g}$ and $H_{max} = \frac{R_{max}}{4}$ at $\theta = 45°$ and initial velocity u $= \frac{R_{max}}{2}$ at $\theta = 90°$ and initial velocity u Change in momentum (i) For completer motion = –2 m u sin θ (ii) at highest point = –m u sin θ $\hat{J}$
Projectile thrown parallel to the horizontal	(a) Equation $y = -\frac{1}{2} g \frac{x^2}{u^2}$ $u_x = u$ $v_x = u$ $u_y = 0$ $v_y = gt$ (downward) $= -gt$ (upward)
	(b) Velocity at any time $v \quad \sqrt{u^2 + g^2t^2}$ $\tan\alpha = \frac{v_y}{v_x}$
	(c) Displacement S = x $\hat{i}$ + y $\hat{J}$ = ut $\hat{i}$ + ½gt² $\hat{J}$
	(d) Time of flight $T = \sqrt{\frac{2h}{g}}$
	(e) Horizontal range $R = u\sqrt{\frac{2h}{g}}$
Projectile thrown from an inclined plane $\vec{a} = -g\sin\theta_0\ \hat{i}$ $\vec{a}_y = -g\cos\theta_0\ \hat{j}$ $\vec{u}_x = u\cos(\theta - \theta_0)\ \hat{i}$ $\vec{u} = u\sin(\theta - \theta_0)\ \hat{j}$	(a) Time of flight $T = \frac{2u_y}{a} = \frac{2u\sin(\theta - \theta_0)}{g\cos\theta_0}$

	$R = u\cos(\theta-\theta_0)T - \frac{1}{2}g\sin\theta_0.T^2$ $R = \frac{2u^2\sin(\theta-\theta_0)\cos\theta}{g\cos^2\theta_0}$ $R = \frac{2u^2\sin(\theta-\theta_0)\cos\theta}{g\cos^2\theta_0}$ Important for R_{max} = $\theta = \frac{\pi}{4} + \frac{\theta_0}{}$ and $R_{max} = \frac{u^2}{g(1+\sin\theta_0)}$
Circular motion	(a) Angle (in radians) $= \frac{\text{arc}}{\text{radius}}$ Or $\Delta\theta = \frac{\Delta S}{r}$ π rad = 180°
	(b) Angular velocity ($\vec{\omega}$) 1. Instantaneous $\omega = \frac{d\theta}{dt}$ 2. Average $\vec{\omega}_{av}$ = $\frac{\text{total angular displacement}}{\text{total time taken}} = \frac{\Delta\theta}{\Delta t}$ If v → linear velocity α → angular acceleration a → linear acceleration
	(c) v = r ω In vector form, $\vec{v} = \vec{\omega} \times \vec{r}$
	(d) $\alpha = \frac{d\vec{\omega}}{dt}$
	(e) a = α r and $\vec{a} = \vec{\alpha} \times \vec{r}$
Newton's equation in circular motion	$\omega = \omega_0 + \alpha t$ $\theta = \omega_0 t + \frac{1}{2} \alpha t^2$ $\omega^2 = \omega_0^2 + 2\alpha\theta$
Centripetal force	$F_c = \frac{mv^2}{r} = m\omega^2 r$ $= m\omega v$
Centripetal acceleration	$a_c = \frac{v^2}{r}$ in vector $\vec{F}_c = m(\vec{v} \times \vec{\omega})$
Total acceleration	$a_T = \sqrt{a^2 + a_c^2}$ a_T → Tangential acceleration a_c → Centripetal acceleration

Motion in horizontal circle	T cos θ = mg T sin θ = mv² / r $\tan\theta = \frac{v^2}{rg}$ $T = mg\sqrt{1 + \frac{v^4}{r^2g^2}}$
	The time period of revolution is $T = 2\pi\sqrt{\frac{h}{g}} = 2\pi\sqrt{\frac{\ell\cos\theta}{}}$
Banking of tracks	$\tan\theta = \frac{v^2}{rg}$, on frictionless road, banked by θ Maximum speed for skidding on a circular unbanked road $v_{max} = \sqrt{\mu rg}$

Top Concepts

- Scalar quantities are quantities with magnitudes only. Examples: Distance, speed, mass, temperature
- Vector quantities are quantities with both magnitude and direction. They obey special rules of vector algebra. Examples: Displacement, velocity, acceleration.
- A vector multiplied by a real number λ is also a vector whose magnitude is dependent on whether λ is positive or negative.
- Two vectors A and B may be added graphically using the head-to-tail method or parallelogram method.
- Vector addition is commutative:

  A + B – B + A

  It also obeys the associative law:

  (A + B) + C = A + (B+C)
- A null or zero vector is a vector with zero magnitude. Because the magnitude is zero, we do not have to specify its direction. It has the properties:

  A + O = A

  λO = O

  OA = O
- The subtraction of vector B from A is defined as the sum of A and –B:

  A – B = A + (−B)

- A vector A can be resolved into components along two given vectors a and b lying in the same plane:

  $A = \lambda a + \mu b$

  where λ and μ are real numbers.

- A unit vector associated with a vector A has magnitude one and is along the vector A:

  $$\hat{n} = \frac{A}{|A|}$$

  The unit vectors $\hat{i}$, $\hat{j}$, $\hat{k}$ are vectors of unit magnitude and point in the direction of the x, y and z axes, respectively, in a right-handed coordinate system.

- Two vectors can be added geometrically by placing them head to tail. The vector connecting the tail of the first to the head of the second is the vector sum or resultant vector.

- Vector R can be resolved into perpendicular components given as R_x and R_y along the x and y axes, respectively.

  $R_x = R\cos\theta$ and $R_y = R\sin\theta$

  An efficient method for adding vectors is using the method of components.

- Unit vectors i, j and k have magnitudes of unity and are directed in the positive direction of the x, y and z axes.

- The position vector of a particle at a particular instant is a vector which goes from the origin of the coordinate system to that point.

- The displacement vector is equal to the final position vector minus the initial position vector.

- Average velocity vector is equal to the change in position vector divided by the corresponding time interval.

- Instantaneous velocity or simply velocity of a particle is along the tangent to the particle's path at each instant.

- Average acceleration is a vector quantity in the same direction as the velocity vector.

- A projectile is an object on which the only force acting is gravity.

- Projectile motion can be thought of as two separate simultaneously occurring components of motion along the vertical and horizontal directions.

- During a projectile's flight, its horizontal acceleration is zero and its vertical acceleration is $-9.8\ m/s^2$.

- The trajectory of a particle in projectile motion is parabolic.

- When a body P moves relative to a body B and B moves relative to A, the velocity of P relative to A is velocity of P relative to B + velocity of P relative to A.

  $$\vec{V}_{P/A} = \vec{V}_{P/B} + \vec{V}_{B/A}$$

  $$\vec{V}_{A\ B} = -\vec{V}_{B/A}$$

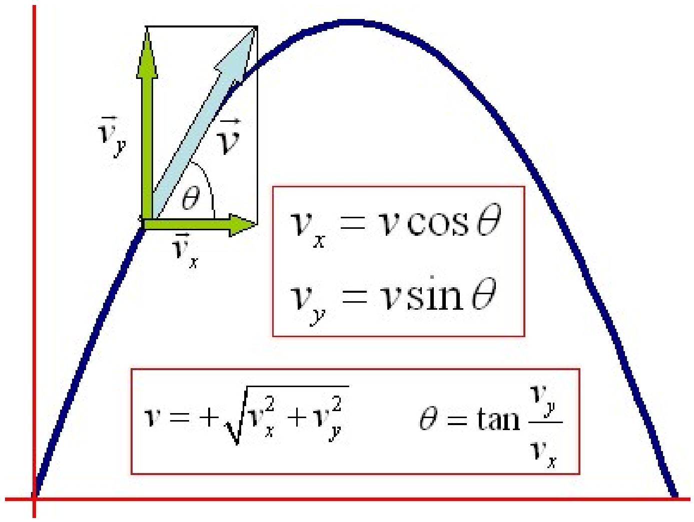
- When an object follows a circular path at constant speed, the motion of the object is called uniform circular motion. The magnitude of its acceleration is $a_c = v^2/R$. The direction of a_c is always towards the center of the circle.

- The angular speed ω is the rate of change of angular distance. It is related to velocity v by $v = \omega R$. The acceleration is $a_c = \omega^2 R$.

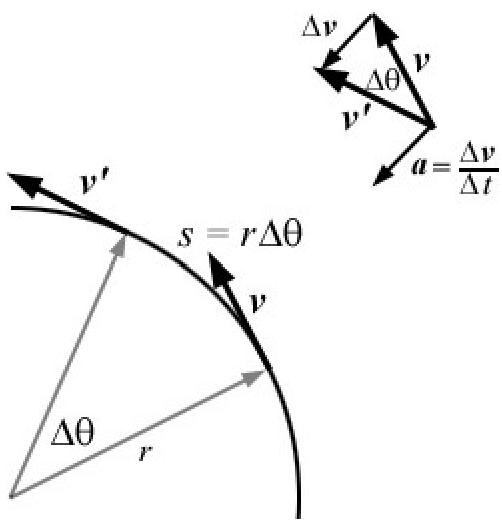
- If T is the time period of revolution of the object in circular motion and ν is the frequency, we have $\omega = 2\pi \nu R$, $a_c = 4\pi^2\nu^2 R$

Diagrams

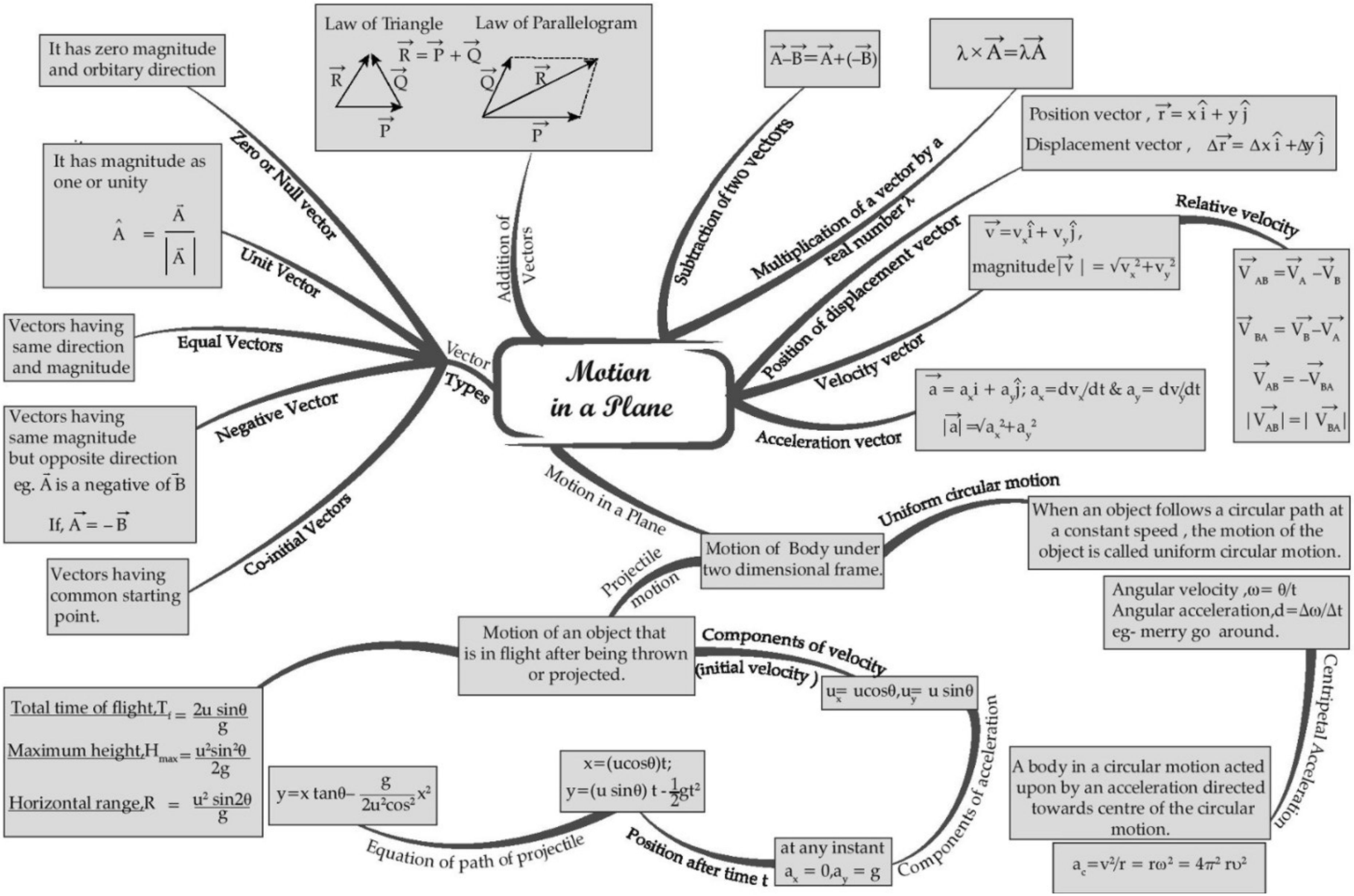
Projectile motion



Velocity and acceleration of an object in uniform circular motion



CHAPTER - 4 : MOTION IN A PLANE



Important Questions

Multiple Choice questions-

1. A body of mass 500 gram is rotating in a vertical circle of radius 1 m. What is the difference in its kinetic energies at the top and the bottom of the circle?

(a) 4.9 J

(b) 19.8 J

(c) 2.8 J

(d) 9.8 J

2. A particle has a displacement of 2 units along the x -axis, 1 unit along the y – axis and 2 units along the z – axis. Then the resultant displacement of the particle is

(a) 3 units

(b) 5 units

(c) 4 units

(d) 1 units

3. A car is moving on a circular path and takes a turn. If R^1 and R^2 are the reactions on the inner and outer wheels respectively, then

(a) $R^1 = > R^2$

(b) $R^1 = R^2$

(c) $R^1 < R^2$

(d) $R^1 > R^2$

4. The angle between centripetal acceleration and tangential acceleration is?

(a) 180°

(b) 0°

(c) 90°

(d) 45°

5. Large angle produces?

(a) high trajectory

(b) curve trajectory

(c) flat trajectory

(d) straight trajectory

6. He dimensional formula for normal acceleration is

(a) LT^{-1}

(b) L^2T^2

(c) L^3T^{-2}

(d) LT^{-2}

7. A book is pushed with an initial horizontal velocity of 5.0 meters per second off the top of a desk. What is the initial vertical velocity of the book?

(a) 10. m/s

(b) 0 m/s

(c) 50 m/s

(d) 2.5 m/s

8. One radian is defined to be the angle subtended where the arc length S is exactly equal to the?

(a) radius of the circle.

(b) diameter of the circle.

(c) circumference of the circle.

(d) half of radius of the circle.

9. A body travels along the circumference of a circle of radius 2 m with a linear velocity of 6 m/s. Then its angular velocity is

(a) 6 rad /s

(b) 3 rad /s

(c) 2 rad / s

(d) 4 rad / s

10. One° (1°) is equal to?

(a) 0.1745 rad

(b) 0.01745 rad

(c) 0.001745 rad

(d) 7.1745 rad

Very Short:

1. Under what condition $|a + b| = |a| + |b|$ holds good?
2. Under what condition $|a - b| = |a| - |b|$ holds good?
3. The sum and difference of the two vectors are equal in magnitude

i. e. $|a + b| = |a - b|$. What conclusion do you draw from this?

4. What is the angle between $\vec{A} \times \vec{B}$ and $\vec{B} \times \vec{A}$?
5. What is the minimum number of coplanar vectors of different magnitudes which can give zero resultant?
6. When $a - b = a + b$ condition holds good than what can you say about b?
7. What is the magnitude of the component of the $9\hat{i} - 9\hat{j} + 19\hat{k}$ vector along the x-axis?
8. Can displacement vector be added to force vector?
9. What is the effect on the dimensions of a vector if it is multiplied by a non-dimensional scalar?
10. (a) What is the angle between $\hat{i} + \hat{j}$ and $\hat{i}$ vectors?

    (b) What is the angle between $\hat{i} - \hat{j}$ and the x-axis?

    (c) What is the angle between $\hat{i} + \hat{j}$ and $\hat{i} - \hat{j}$?ss

Short Questions:

1. Name two quantities that are the largest when the maximum height attained by the projectile is largest.
2. A stone dropped from the window of a stationary railway carriage takes 2 seconds to reach the ground. At what time the stone will reach the ground when the carriage is moving with

   (a) the constant velocity of $80 kmh^{-1}$

   (b) constant acceleration of $2 ms^{-2}$?
3. Can a particle accelerate when its speed is constant? Explain.
4. (a) Is circular motion possible at a constant speed or at constant velocity? Explain.

   (b) Define frequency and time period.
5. When the component of a vector A along the direction of vector B is zero, what can you conclude about the two vectors?
6. Comment on the statement whether it is true or false “Displacement vector is fundamentally a position vector.’’ Why?
7. Does the nature of a vector changes when it is multiplied by a scalar?
8. Can the walk of a man be an example of the resolution of vectors? Explain.

Long Questions:

1. Discuss the problem of a swimmer who wants to cross the river in the shortest time.
2. State and prove parallelogram law of vector addition. Discuss some special cases.

3. Derive the relation between linear velocity and angular velocity. Also, deduce its direction.

Assertion Reason Questions:

1. **Directions:** Each of these questions contain two statements, Assertion and Reason. Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select one of the codes (a), (b), (c) and (d) given below.

   (a) Assertion is correct, reason is correct; reason is a correct explanation for assertion.
   (b) Assertion is correct, reason is correct; reason is not a correct explanation for assertion
   (c) Assertion is correct, reason is incorrect
   (d) Assertion is incorrect, reason is correct.

   Assertion: In projectile motion, the angle between the instantaneous velocity and acceleration at the highest point is 180°.
   Reason: At the highest point, velocity of projectile will be in horizontal direction only.

2. **Directions:** Each of these questions contain two statements, Assertion and Reason. Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select one of the codes (a), (b), (c) and (d) given below.

   (a) Assertion is correct, reason is correct; reason is a correct explanation for assertion.
   (b) Assertion is correct, reason is correct; reason is not a correct explanation for assertion
   (c) Assertion is correct, reason is incorrect
   (d) Assertion is incorrect, reason is correct.

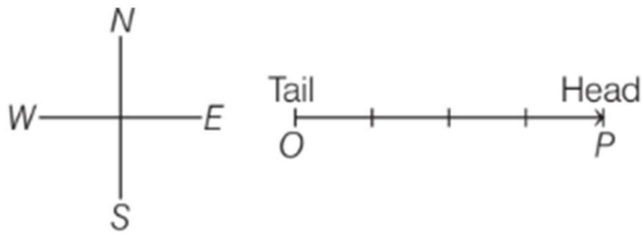
   Assertion: Two particles of different mass, projected with same velocity at same angles. The maximum height attained by both the particle will be same.
   Reason: The maximum height of projectile is independent of particle mass.

Case Study Questions:

1. Vectors are the physical quantities which have both magnitudes and directions and obey the triangle/parallelogram laws of addition and subtraction. It is specified by giving its magnitude by a number and its direction. e.g., Displacement, acceleration, velocity, momentum, force, etc. A vector is represented by a bold face type and also by an arrow placed over a letter, i.e.

   $\mathbf{F}, \mathbf{a}, \mathbf{b}$ or $\vec{F}, \vec{a}, \vec{b}.$

   The length of the line gives the magnitude, and the arrowhead gives the direction. The point P is called head or terminal point and point O is called tail or initial point of the vector **OP**.



i. Amongst the following quantities, which is not a vector quantity?

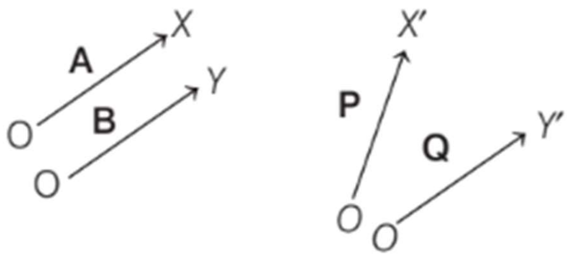
(a) Force

(b) Acceleration

(c) Temperature

(d) Velocity

ii. Set of vectors A and B, P and Q are as shown below



Length of A and B is equal, similarly length of P and Q is equal. Then, the vectors which are equal, are

(a) A and P

(b) P and Q

(c) A and B

(d) B and Q

iii. $|\lambda A| = \lambda |A|$, if

(a) $\lambda > 0$

(b) $\lambda < 0$

(c) $\lambda = 0$

(d) $\lambda \neq 0$

iv. Among the following properties regarding null vector which is incorrect?

(a) A + 0 = A

(b) $\lambda 0 = \lambda$

(c) 0A = 0

(d) A – A = 0

v. The x and y components of a position vector P have numerical values 5 and 6, respectively. Direction and magnitude of vector P are

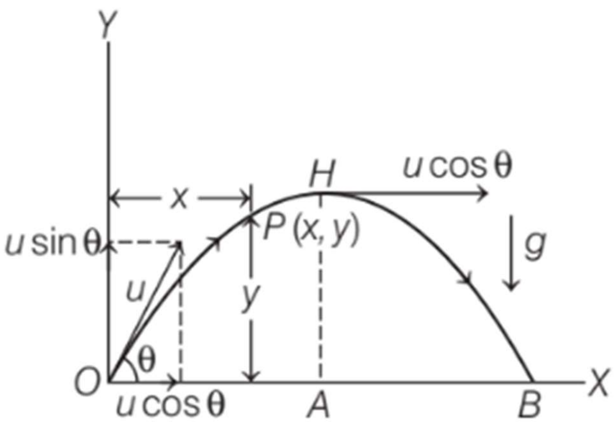
(a) $\tan^{-1}\left(\frac{6}{5}\right)$ and $\sqrt{61}$

(b) $\tan^{-1}\left(\frac{5}{6}\right)$ and $\sqrt{61}$

(c) 60° and 8

(d) 30° and 9

2. Projectile motion is a form of motion in which an object or particle is thrown with some initial velocity near the earth's surface, and it moves along a curved path under the action of gravity alone. The path followed by a projectile is called its trajectory, which is shown below. When a projectile is projected obliquely, then its trajectory is as shown in the figure below



Here velocity u is resolved into two components, we get (a) u cos θ along OX and (b) u sin θ along OY

i. The example of such type of motion is

(a) Motion of car on a banked road

(b) Motion of boat in sea

(c) A javelin thrown by an athlete

(d) Motion of ball thrown vertically upward

ii. The acceleration of the object in horizontal direction is

(a) Constant

(b) Decreasing

(c) Increasing

(d) Zero

iii. The vertical component of velocity at point H is

(a) Maximum

(b) Zero

(c) Double to that at O

(d) Equal to horizontal component

iv. A cricket ball is thrown at a speed of 28 m/s in a direction 30° with the horizontal. The time taken by the ball to return to the same level will be

(a) 2.0 s

(b) 3.0 s

(c) 4.0 s

(d) 2.9 s

v. In above case, the distance from the thrower to the point where the ball returns to the same level will be

(a) 39 m

(b) 69 m

(c) 68 m

(d) 72 m

✓ Answer Key:

Multiple Choice Answers-

1. Answer: (d) 9.8 J
2. Answer: (a) 3 units
3. Answer: (c) $R^1 < R^2$
4. Answer: (c) 90°
5. Answer: (a) high trajectory
6. Answer: (d) LT^{-2}
7. Answer: (b) 0 m/s
8. Answer: (a) radius of the circle.
9. Answer: (b) 3 rad /s
10. Answer: (b) 0.01745 rad

Very Short Answers:

1. Answer: When a and b act in the same direction i. e. when 0 = 0 between • them, then |a + b|=|a| + |b|.
2. Answer: The condition |a – b|=|a| – |b| holds goods when a and b act in the opposite

direction.

3. Answer: The two vectors are equal in magnitude and are perpendicular to each other.
4. Answer: The given vectors act along two parallel lines in opposite directions i.e. they are anti-parallel, so the angle between them is 180°.
5. Answer: 3, If three vectors can be represented completely by the three sides of a triangle taken in the same order, then their resultant is zero.
6. Answer: For a – b = a + b condition to hold good, b must be a null vector.
7. Answer: 9.
8. Answer: No.
9. Answer: There is no effect on the dimensions of a vector if it is multiplied by a non-dimensional scalar.
10. Answer:

(a) 45°

(b) 45°

(c) 90°

Short Questions Answers:

1. Answer: Time of flight and the vertical component of velocity are the two quantities that are the largest when the maximum height attained by the projectile is the largest.
2. Answer: The time taken by the freely falling stone to reach the ground is given by

$t = \sqrt{\frac{2h}{g}}$

In both cases, the stone will fall through the same height as it is falling when the railway carriage is stationary. Hence the stone will reach the ground after 2 seconds.

3. Answer: Yes. A particle can be accelerated if its velocity changes. A particle having uniform circular motion has constant speed but its direction of motion changes continuously. Due to this, there is a change in velocity and hence the particle is moving with variable velocity. Thus, particle is accelerating.
4. Answer:

(a) Circular motion is possible at a constant speed because, in a circular motion, the magnitude of the velocity i.e. speed remains constant while the direction of motion changes continuously.

(b) Frequency is defined as the number of rotations completed by a body in one second and the time period is defined as the time taken by an object to complete one rotation.

5. Answer:

The two vectors A and B are perpendicular to each other.

Explanation: Let θ = angle between the two vectors A and B component of vector A along the direction of B is obtained by resolving A i.e. A cos θ.

Now according to the statement

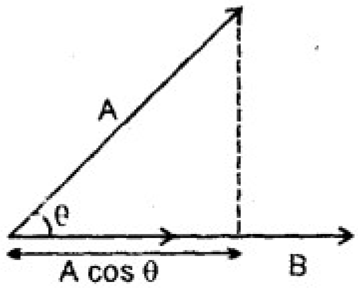
$A \cos \theta = 0$

or

$\cos \theta = 0 = \cos 90°$

$\theta = 90°$

i.e. $A \perp B$



Hence proved.

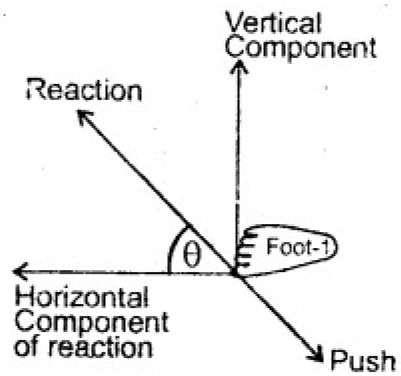
6. Answer: The given statement is true. The displacement vector gives the position of a point just like the position vector. The only difference between the displacement and the position vector is that the displacement vector gives the position of a point with reference to a point other than the origin, while the position vector gives the position of a point with reference to the origin. Since the choice of origin is quite arbitrary, so the given statement.

7. Answer: The nature of a vector may or may not be changed when it is multiplied.

   For example, when a vector is multiplied by a pure number like 1, 2, 3, …. etc., then the nature of the vector does not change. On the other hand, when a vector is multiplied by a scalar physical quantity, then the nature of the vector changes.

   For example, when acceleration (vector) is multiplied by a mass (scalar) of a body, then it gives force (a vector quantity) whose nature is different than acceleration.

8. Answer: Yes, when a man walks, he pushes the ground with his foot. In return, an equal and opposite reaction acts on his foot. The reaction is resolved into two components: horizontal and vertical components. The horizontal component of the reaction helps the man to move forward while the vertical component balances the weight of the man.

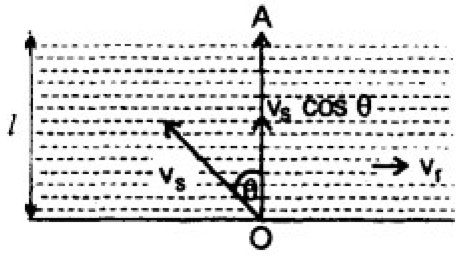


Long Questions Answers:

1. Answer:

Let v_s and v_r be the velocities of swimmer and river respectively.

Let v = resultant velocity of vs and v_r



1. Let the swimmer begins to swim at an angle θ with the line OA where OA is ⊥ to the flow of the river.

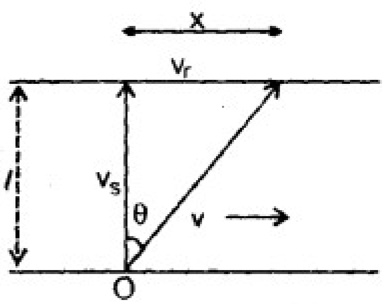
If t = time taken to cross the river, then

$t = \frac{l}{v_s \cos\theta}$ ...(i)

where l = breadth of the river

For t to be minimum, cos 0 should be maximum.

i.e., cos θ = 1



This is possible if θ = 0

Thus, we conclude that the swimmer should swim in a direction perpendicular to the direction of the flow of the river.

2. v = $\sqrt{v_s^2 + v_r^2}$

where v$\vec{v}$ is the resultant velocity of v_s and v_r.

3. tan θ = $\frac{v_r}{v_s} = \frac{x}{l}$

or

x = l$\frac{v_r}{V_s}$

4. t = $\frac{l}{V_s}$

2. Answer:

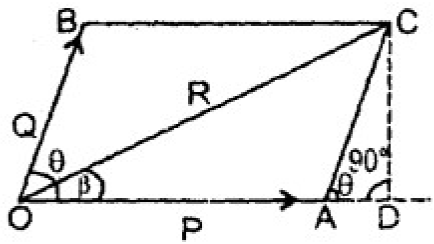
It states that if two vectors can be represented completely (i.e. both in magnitude and direction) by the two adjacent sides of a parallelogram drawn from a point then their resultant is represented completely by its diagonal drawn from the same point.

Proof: Let P and Q be the two vectors represented completely by the adjacent sides OA and OB of the parallelogram OACB s.t.

$\overrightarrow{OA}$ = P, $\overrightarrow{OB}$ = Q

or

| $\overrightarrow{OA}$ | = |P|, | $\overrightarrow{OB}$ | = |Q|



θ = angle between them = ∠AOB

If R be their resultant, then it will be represented completely by the diagonal OC through point O s.t. OC = R

The magnitude of R: Draw CD ⊥ to OA produced,

$$\therefore \quad \angle DAC = \angle AOB = \theta$$

Now in right angled triangle ODC,

$$OC^2 = OD^3 + DC^2$$
$$= (OA + AD)^2 + DC^2$$
$$= OA^2 + AD^2 + 2.OA.AD + DC^2$$
$$= OA^2 + (AD^2 + DC^2) + 2OA.AD \quad \text{.... }(i)$$

Also in r.t. ∠d ΔADC,

$$AC^2 = AD^2 + DC^2 \quad \text{.... }(ii)$$

Also $$\frac{AD}{AC} = \cos\theta$$

or $$AD = AC\cos\theta \quad \text{... }(iii)$$

and $$\frac{DC}{AC} = \sin\theta$$

or $$DC = AC\sin\theta \quad \text{.... }(iv)$$

∴ from equation (*i*), (*ii*), (*iii*), we get

$$OC^2 = OA^2 + AC^2 + 2.OA.AC\cos\theta$$

or $$OC = \sqrt{OA^2 + AC^2 + 2.OA.AC\cos\theta} \quad \text{.... }(v)$$

As OC = R, OA = P, AC = OB = Q .... (*vi*)

∴ from (v) and (*vi*), we get

$$R = \sqrt{P^2 + Q^2 + 2PQ\cos\theta} \quad \text{.... }(vii)$$

eqn. (vii) gives the magnitude of R.

The direction of R: Let β be the angle made by R with P

∴ in rt. ∠d ΔODC,

$$\tan\beta = \frac{DC}{OD} = \frac{DC}{OA + AD}$$

$$= \frac{AC\sin\theta}{OA + AC\cos\theta} \quad \text{[by using (iii) and (iv)]}$$

$$\tan\beta = \frac{Q\sin\theta}{P + Q\cos\theta} \quad \text{... }(viii)$$

Special cases: (a) When two vectors are acting in the same direction:

Then θ = 0°

∴ $R = \sqrt{(P+Q)^2} = P + Q$

and $\tan\beta = \frac{Q.0}{P+Q} = 0 \text{ or } \beta = 0°$

Thus, the magnitude of the resultant vector is equal to the sum of the magnitudes of the two vectors acting in the same direction, and their resultant acts in the direction of P and Q.

(b) When two vectors act in the opposite directions:

Then θ = 180°

∴ $\cos\theta = -1 \text{ and } \sin\theta = 0$

∴ $R = \sqrt{P^2 + Q^2 + 2PQ(-1)}$

$= \sqrt{P^2 + Q^2 - 2PQ}$

$= \sqrt{(P-Q)^2} \text{ or } \sqrt{(Q-P)^2}$

$= (P - Q) \text{ or } (Q - P)$

and $\tan\beta = \frac{Q\times 0}{P+Q(-1)} = 0 = \tan 0° \text{ or } \tan 180°$

∴ $\beta = 0° \text{ or } 180°$

Thus, the magnitude of the resultant of two vectors acting in the opposite direction is equal to the difference of the magnitude of two vectors and it acts in the direction of the bigger vector.

(c) If θ = 90° i.e. if P ⊥ Q,

then cos 90° = 0

and

sin 90° = 1

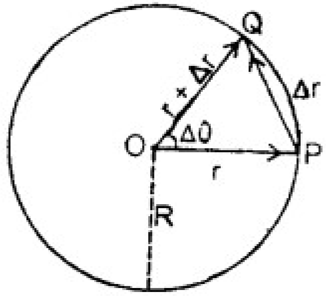
R = $\sqrt{P^2 + Q^2}$

and

tan β = $\frac{Q}{P}$

3. Answer:

Let R be the radius of the circular path of centre O on which an object is moving with uniform angular velocity co. Let v = its linear velocity. Let the object move from point P at time t to point Q at time t + Δt. If r and r + Δr be its position vectors at point P and Q respectively, then



$$\vec{OP} = \mathbf{r}$$

and $$\vec{OQ} = \mathbf{r} + \Delta\mathbf{r}$$

Also $$|\mathbf{r}| = |\mathbf{r} + \Delta\mathbf{r}| = R$$
$$= \text{radius of circle.}$$

∴ Linear displacement of the particle from P to Q in small time interval Δt = Δr.

Let Δθ = its angular displacement

∴ $\omega = \frac{\Delta\theta}{\Delta t}$

or

Δθ = ωΔt ….(1)

Also we know that $\Delta\theta = \frac{\widehat{PQ}}{R}$ …(2)

∴ from (1) and (2), we get

$$\omega\Delta t = \frac{\widehat{PQ}}{R} \text{ or } \frac{\widehat{PQ}}{\Delta t} = \omega R \quad .... (3)$$

Now when Δt → 0, then from eqn. (1) Δθ → 0

so arc PQ = $\widehat{PQ}$ = chord PQ

Thus eqn. (3) reduces to

$$\frac{PQ}{\Delta t} = \omega R$$

or $$v = R\omega$$

where $v = \frac{PQ}{\Delta t}$ is the linear velocity of the object.

Direction of velocity vector: In isosceles ΔOPQ,

$$\angle PQO + \angle OPQ + \angle QOP = 180^\circ$$

As $\angle Q = \angle P$

$\therefore$ $2\angle P + \Delta\theta = \pi$

or $\angle QPO = \frac{\pi}{2} - \frac{\Delta\theta}{2}$

$$= \frac{\pi}{2} - \frac{\omega\Delta t}{2} \quad .... (4)$$

when $\Delta t \to 0$, $\angle QPO \to \frac{\pi}{2}$

i.e. $\overrightarrow{OP}$ tends to become ⊥ to $\overrightarrow{OP}$

or

$\overrightarrow{OP}$ tends to lie along the tangent at P. Hence velocity vector at P is directed along the tangent to the circle in the direction of motion

Assertion Reason Answer:

1. (d) Assertion is incorrect, reason is correct.
2. (a) Assertion is correct, reason is correct; reason is a correct explanation for assertion.

Case Study Answer:

1. i (c) Temperature

Explanation:

Temperature is not a vector quantity because it has magnitude only. However, force, acceleration and velocity have both a magnitude and a direction. So, these are vectors in nature.

ii (c) A and B

Explanation:

Two vectors are said to be equal, if and only if they have the same magnitude and direction. Among the given vectors A and B are equal vectors as they have same magnitude (length) and direction. However, P and Q are not equal even though they are of same magnitude because their directions are different.

iii (a) $\lambda > 0$

Explanation:

$|\lambda$ A | = $\lambda\,|A|$, if $\lambda > 0$ as multiplication of vector A with a positive number λ gives a vector whose magnitude is changed by the factor λ but the direction is same as that of A.

iv (b) $\lambda 0 = \lambda$

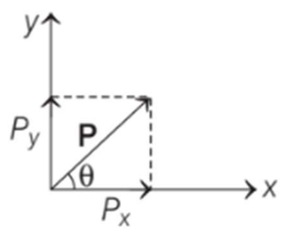
Explanation:

Null vector 0 is a vector, whose magnitude is zero and its direction cannot be specified. So, it means, |0|= 0. Thus, λ 0 = 0. Hence, property given in option (b) is incorrect.

v (a) $\tan^{-1}\left(\frac{6}{5}\right)$ and $\sqrt{61}$

Explanation:

Let **P** be as shown in the figure, then according to the given information



$$P_x = 5,\ P_y = 6$$

$$\therefore \quad |\mathbf{P}| = \sqrt{P_x^2 + P_y^2} = \sqrt{25 + 36}$$

$$\Rightarrow \quad |\mathbf{P}| = \sqrt{61}$$

and $\tan\theta = \frac{P_y}{P_x} = \frac{6}{5} \Rightarrow \theta = \tan^{-1}\left(\frac{6}{5}\right)$

2. i (c) a javelin thrown by an athlete

Explanation:

A javelin thrown by an athlete is an example of projectile motion.

ii (d) zero

Explanation:

The horizontal component of velocity (u cos θ) is constant throughout the motion, so there will be no acceleration in horizontal direction.

iii (b) zero

Explanation:

As the vertical components of velocity (u sin θ) decreases continuously with height, from O to H, due to downward force of gravity and becomes zero at H

iv (d) 2.9 s

Explanation:

The time taken by the ball to return to the same level,

$$T = \frac{2v_0 \sin\theta}{g} = \frac{2 \times 28 \times \sin 30^\circ}{9.8} \approx 2.9 \text{ s}$$

v (b) 69 m

Explanation:

The distance from the thrower to the point where the ball returns to the same level is

$$R = \frac{v_0^2 \sin 2\theta}{g} = \frac{28 \times 28 \times \sin 60^\circ}{9.8} \approx 69 \text{ m}$$